A study on geometrical curves based on their composition

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Abstract: In the plastic sense, the 3-dimensional objects given by geometrical curves represent a very interesting phenomenon. I have studied their form in my earlier work, using simple geometrical curves with a given defining law, then constructing variations of various forms in the three dimensional Euclidean space and, finally representing them in a two dimensional space. Based on the above, we shall consider new parameters grounded on two concepts, the form composition and the color composition, to construct variations for both. This paper focuses on geometrical curves, using computer graphics, and represents them in an attempt to show different variations of their composition, all of which are interesting both in the visual and the plastic sense.

Key words: geometrical curve, form composition, color composition

1.Introduction

In the plastic sense, the 3-dimensional objects given by geometrical curves represent a very interesting phenomenon. I have studied their form in my earlier work [1,2,3], using simple geometrical curves with a given defining law, then constructing variations of various forms in the three dimensional Euclidean space and, finally converting them in a two dimensional object. Based on the above, we shall consider new parameters grounded on two concepts, the form composition and the color composition, to construct variations for both. We start considering three parameters on the form composition: First, regarding the type of geometrical curves, we use the epicycloid, the hypocycloid, the epitrochoid, the hypotrochoid, and the torus knot. Secondly, regarding the composition, we use composition with a unit and the composition with similar figures. Thirdly, in the law of the configuration, we use the composition via rotational symmetry and a helical motion. We also consider three options regarding the color composition. First, we use a color scheme with the same hue. Secondly, we use a color scheme with gradation of hues. Thirdly, we use a color scheme with contrasting hues.

This paper focuses on the concepts of form composition and color composition and, by using computer graphics, we attempt to show variations of the composition of geometrical curves, which are interesting in both the visual and the plastic sense.

2. Parameters for the compositions used

2.1 Parameters for the form composition

The parameters of the form composition are as follows:

(1)For the type of geometrical curves, we use the epicycloid, the hypocycloid, the epitrochoid, the hypotrochoid, and the torus knot. (2)For the classification of the composition, we use composition with a unit and the composition with the similar figures. (3)For the classification of the thickness of the line, we set the thickness of the line uniformly in case of the composition with a unit, otherwise the thickness of the line is changed geometrically. (4)For the defining law of the configuration, we use the composition based on the rotational symmetry, and the parameters of the helical motion.

2.2 Conditions for the color composition

In this section we will consider the following three conditions in term of the color composition. The conditions of the color composition are as follows;

(1)We applied the color scheme with the similar hue. (2)We applied the color scheme with the gradation of hue. (3)We applied the color scheme with the contrast of hue.

2.2 Conditions for the color composition

The parameters for the color composition are as follows;

(1)A color scheme with the same hue. (2) A color scheme with a gradation of hues. (3) A color scheme with contrasting hues.

2.3 Parameters for constructing the composition

Regarding the parameters for constructing a composition this paper:

(1) represents the composition for each model.

(2) uses a number for each of the geometrical curves, the angle of rotation and, the scale ratio. For example, Fig. 10(b)(4, 30°) and Fig 10(c)(4, 15°, 0.88).

3. Composition of the geometrical curves

3.1 Composition of cycloid curves

3.1.1 Composition of epicycloid curves

Epicycloid curves can be defined as follws by parameter representation[1].

$$x = (a+b)\cos t - b\cos (a+b)\, t / b$$
$$y = (a+b)\sin t - b\sin (a+b)\, t / b \qquad (1)$$

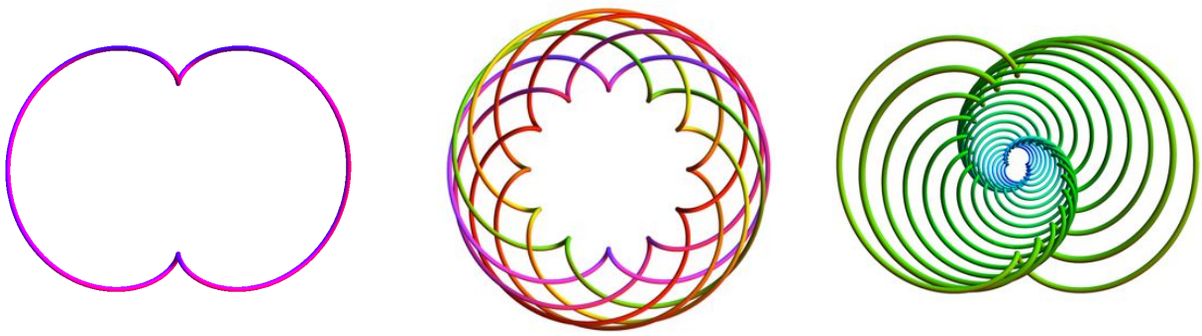
Adding the cordinate z to (1), it becomes:

$$x = (a+b)\cos t - b\cos (a+b)\, t / b$$
$$y = (a+b)\sin t - b\sin (a+b)\, t / b$$
$$z = e \sin f\, t \qquad (2)$$

The geometrical curves are obtained from (2) by varying parameters.

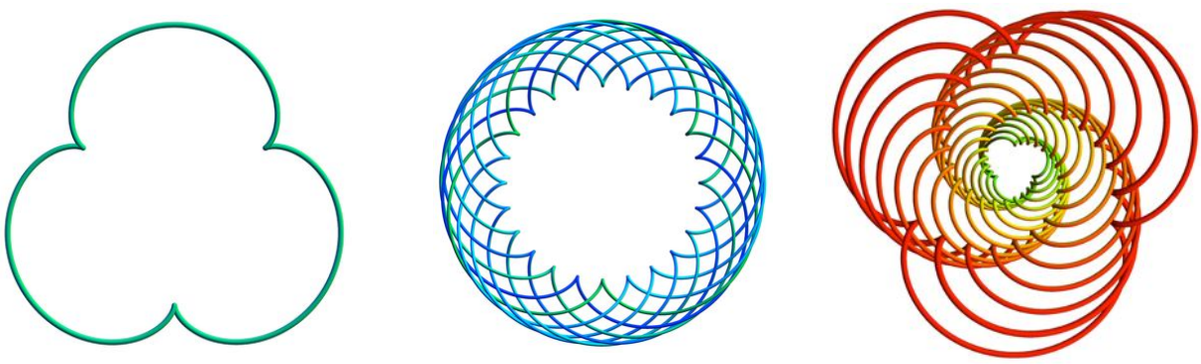
Examples of compositions, obtained from epicycloid curves, are shown in Fig. 1-10.

Fig. 1(b) was constructed with the units of Fig. 1(a), and the color scheme was applied with a contrast of hues. Fig. 1(c) was formed with via the combination of similar figures based on Fig. 1(a), and the color scheme was applied with a gradation of hues. Fig. 2(b) was constructed with the units of Fig. 2(a), and the color scheme was applied with the same hue. Fig. 2(c) was formed with via the combination of the similar figures based on Fig. 2(a), and the color scheme was applied with a gradation of hues. Fig. 3(b) was constructed with the units of Fig. 3(a), and the color scheme was applied with a contrast of hues. Fig. 3(c) was formed with via the combination of the similar figures based on Fig. 3(a), and the color scheme was applied with a gradation of hues. Fig. 4(b) was constructed with the units of Fig. 4(a), and the color scheme was applied with the same hue. Fig. 4(c) was formed with via the combination of the similar figures based on Fig. 4(a), and the color scheme was applied with a gradation of hues. Fig. 5(b) was constructed with the units of Fig. 5(a), and the color scheme was applied with the same hue. Fig. 5(c) was formed with via the combination of the similar figures based on Fig. 5(a), and the color scheme was applied with a gradation of hues.



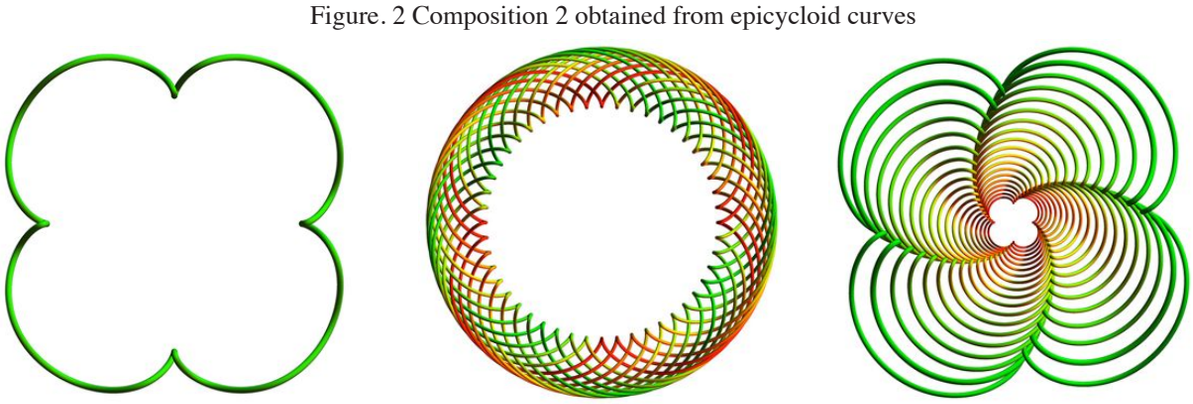
(a) (b) (6, 30°) (c) (19, 15°, 0.88)

Figure. 1 Composition 1 obtained from epicycloid curves



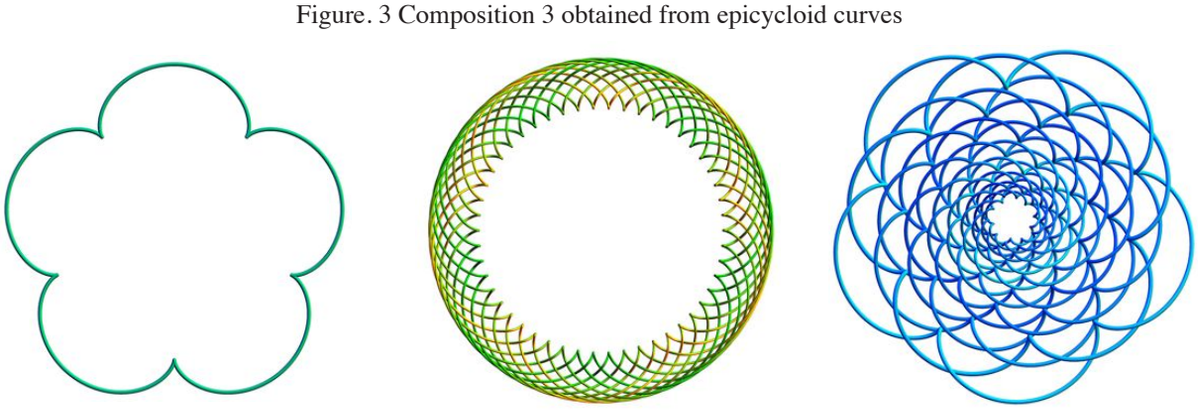
(a) (b) (8, 15°) (c) (17, 15°, 0.9)

Figure. 2 Composition 2 obtained from epicycloid curves



(a) (b) (12, 7.5°) (c) (19, 7.5°, 0.9)

Figure. 3 Composition 3 obtained from epicycloid curves



(a) (b) (10, 7.2°) (c) (19, 15°, 0.9)

Figure. 4 Composition 4 obtained from epicycloid curves

(a) (b) (8, 7.5°) (c) (22, 15°, 0.9)

Figure. 5 Composition 5 obtained from epicycloid curves

3.1.2 Composotion of hypocycloid curves

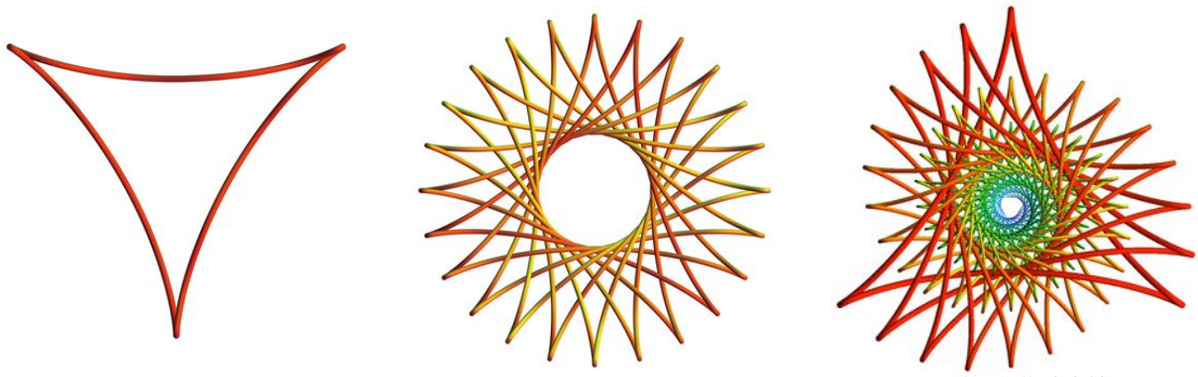
Hypocycloid curves are defined by the following parametric representation:

$$x = (a-b)\cos t + b\cos (a-b)\, t / b$$
$$y = (a-b)\sin t - b\sin (a-b)\, t / b \quad (3)$$

Adding the cordinate z to (3), it becomes:

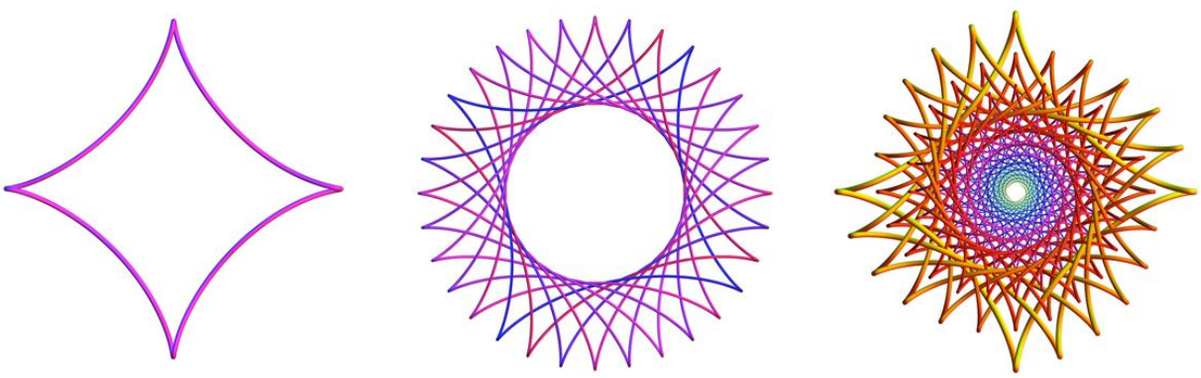
$$x = (a-b)\cos t + b\cos (a-b)\, t / b$$
$$y = (a-b)\sin t - b\sin (a-b)\, t / b$$
$$z = e \sin f\, t \quad (4)$$

The geometrical curves are obtained from (4) by varying parameters. Examples of compositions, obtained from hypocycloid curves, are shown in Fig. 6-15. Fig.6(b) uses the units of Fig. 6(a), and the color scheme applied was with the same hue. Fig. 6(c) was formed via the combination of similar figures based on Fig. 6(a), and the color scheme applied was a gradation of hues. Fig. 7(b) uses the units of Fig. 7(a), and the color scheme applied was with the same hue. Fig. 7(c) was formed with via the combination of the similar figures based on Fig. 7(a), and the color scheme applied was a gradation of hues. Fig. 8(b)uses the units of Fig. 8(a), and the color scheme applied was with he same hue. Fig. 8(c) was formed via the combination of the similar figures based on Fig.8(a), and the color scheme applied was a gradation of hues. Fig. 9(b) uses the units of Fig. 9(a), and the color scheme applied was a contrast of hues. Fig. 9(c) was formed via the combination of the similar figures based on Fig. 9(a), and the color scheme applied was a gradation of hues. Fig. 10(b) uses the units of Fig. 10(a), and the color scheme applied was with the same hue. Fig. 10(c) was formed via the combination of the similar figures based on Fig. 10(a), and the color scheme applied was with a gradation of hues. Fig. 11(b) uses the units of Fig. 11(a), and the color scheme applied was with the same hue. Fig. 11(c) was formed via the combination of the similar figures based on Fig. 11(a), and the color scheme applied was a gradation of hues. Fig. 12(b) uses the units of Fig. 12(a), and the color scheme applied was with the same hue. Fig. 12(c) was formed visa the combination of the similar figures based on Fig. 12(a), and the color scheme applied was a gradation of hues. Fig. 13(b) uses the units of Fig. 13(a), and the color scheme applied was with the same hue. Fig. 13(c) was formed via the combination of the similar figures based on Fig. 13(a), and the color scheme applied was a gradation of hues. Fig. 14(b) uses the units of Fig. 14(a), and the color scheme applied was with the same hue. Fig. 14(c) was formed via the combination of the similar figures based on Fig. 14(a), and the color scheme applied was a gradation of hues. Fig. 15(b) uses the units of Fig. 15(a), and the color scheme applied was with the same hue. Fig. 15(c) was formed via the combination of the similar figures based on Fig. 15(a), and the color scheme applied was a gradation of hues.



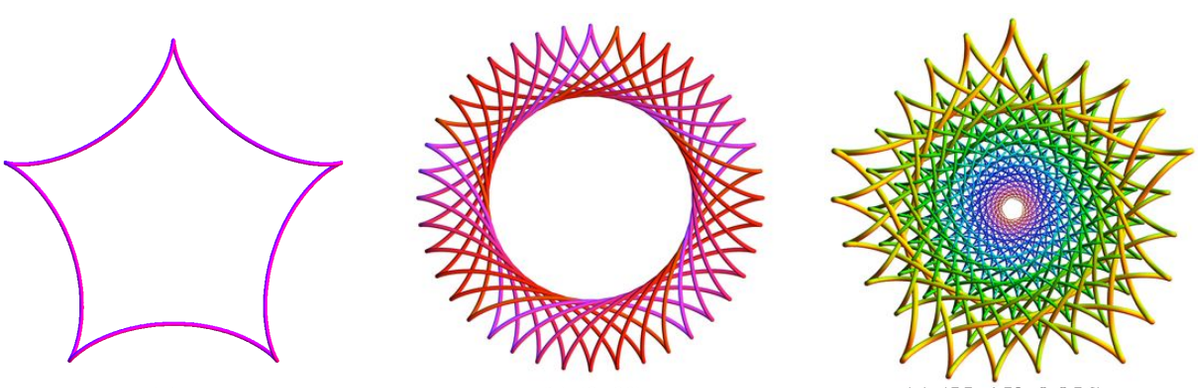
(a) (b) (8, 15°) (c) (37, 15°, 0.948)

Figure. 6 Composition 1 obtained from hypocycloid curves



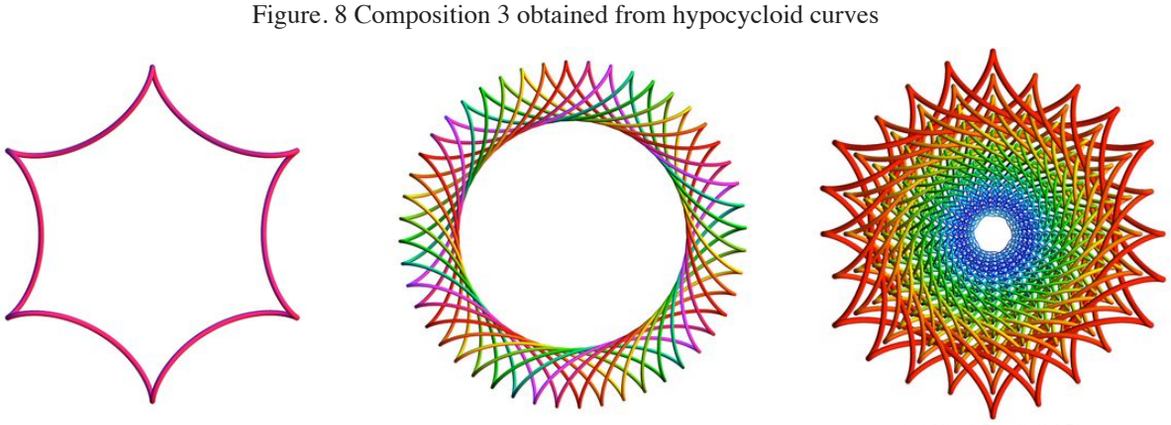
(a) (b) (8, 11.25°) (c) (55, 15°, 0.958)

Figure. 7 Composition 2 obtained from hypocycloid curves



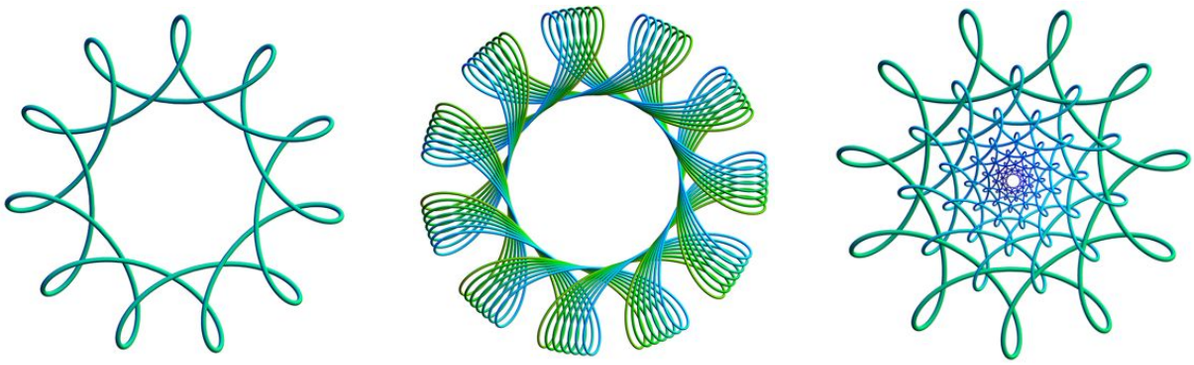
(a) (b) (8, 9°) (c) (55, 15°, 0.956)

Figure. 8 Composition 3 obtained from hypocycloid curves



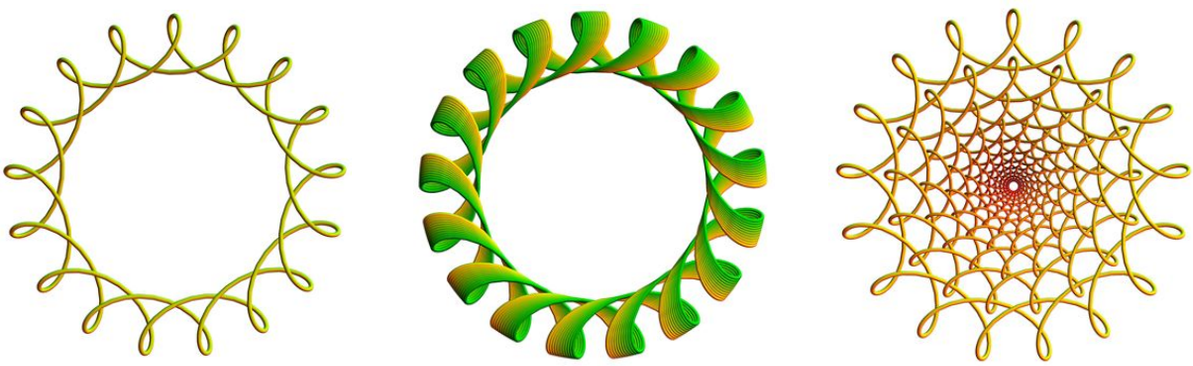
(a) (b) (9, 7.5°) (c) (60, 15°, 0.965)

Figure. 9 Composition 4 obtained from hypocycloid curves



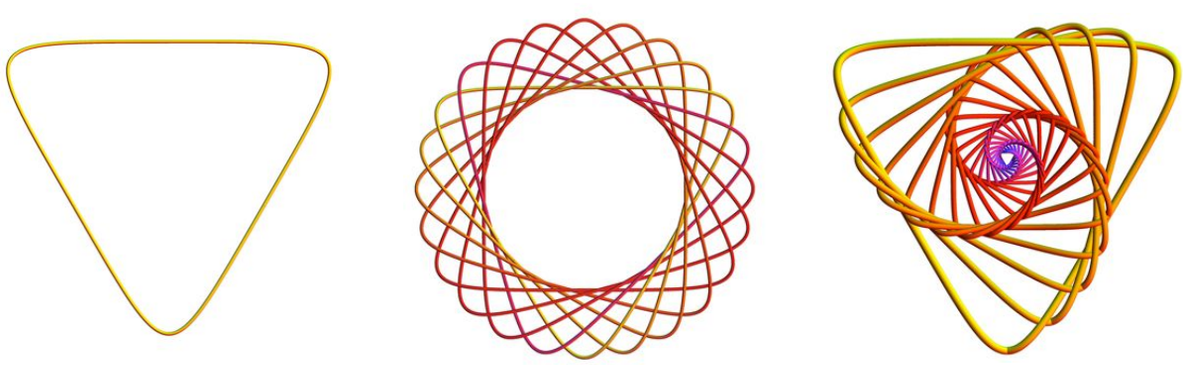
(a) (b) (8, 15°) (c) (8, 15°, 0.65)

Figure. 10 Composition 5 obtained from hypocycloid curves



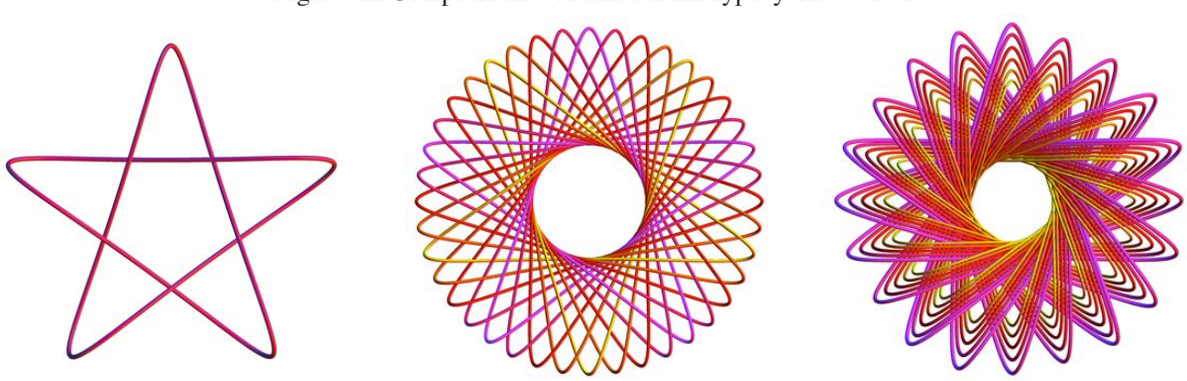
(a) (b) (11 3.53°) (c) (11, 5.29°, 0.73)

Figure. 11 Composition 6 obtained from hypocycloid curves



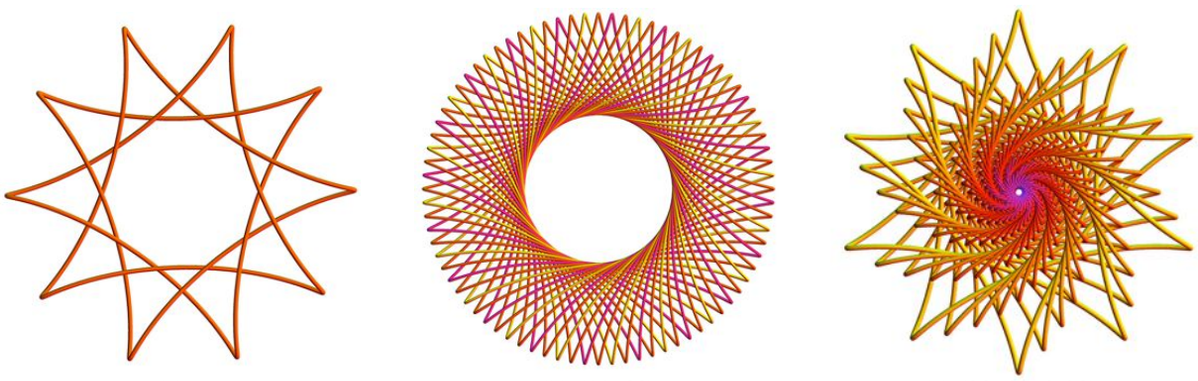
(a) (b) (8, 15°) (c) (24, 15°, 0.88)

Figure. 12 Composition 7 obtained from hypocycloid curves



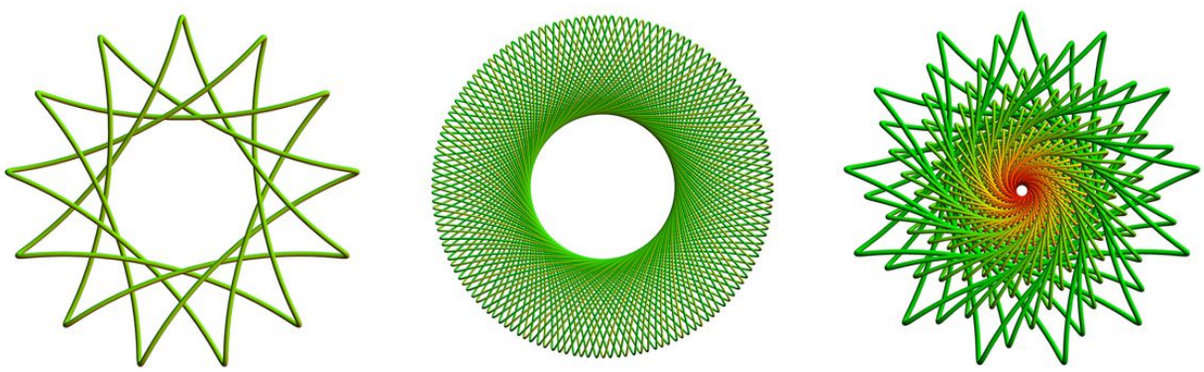
(a) (b) (8, 18°) (c) (21, 18°, 0.98)

Figure. 13 Composition 8 obtained from hypocycloid curves



(a) (b) (11, 9°) (c) (22, 9°, 0.88)

Figure. 14 Composition 9 obtained from hypocycloid curves



(a) (b) (11, 7.5°) (c) (21, 9°, 0.88)

Figure. 15 Composition 10 obtained from hypocycloid curves

3.1.3 Composotion of epitrochoid curves

Epitrochoid curves are defined by the following parametric representation:

$$x = (a+b)\cos t - c\cos (a+b)\, t / b$$
$$y = (a+b)\sin t - c\sin (a+b)\, t / b \qquad (5)$$

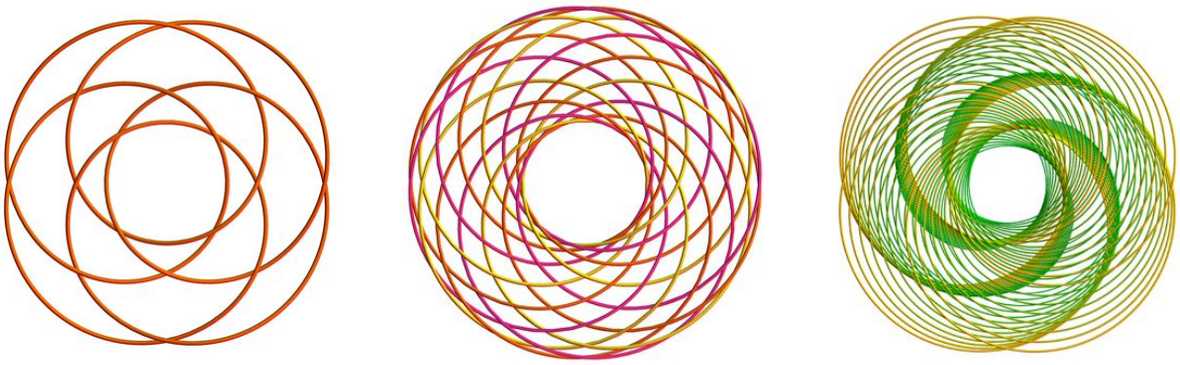
Adding the cordinate z to (5), it becomes:

$$x = (a+b)\cos t - c\cos (a+b)\, t / b$$
$$y = (a+b)\sin t - c\sin (a+b)\, t / b$$
$$z = e\sin f\, t \qquad (6)$$

The geometrical curves are obtained from (6) by varying parameters.

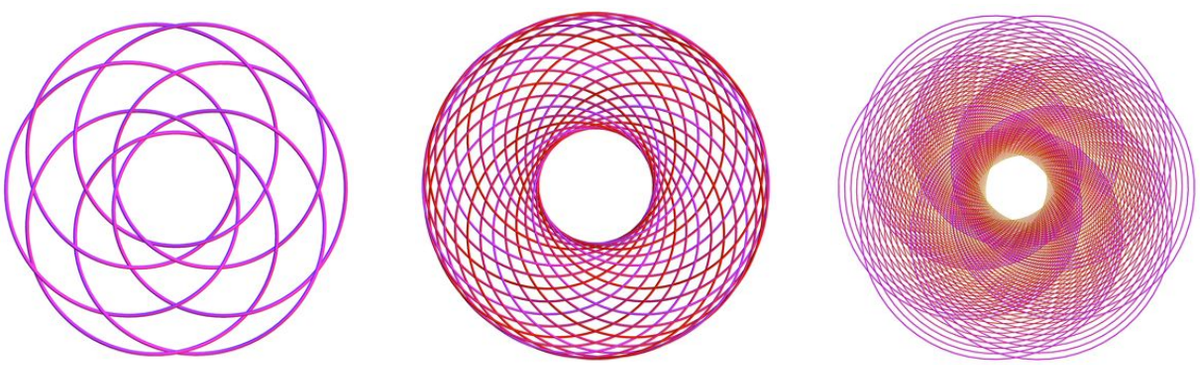
Examples of compositions, obtained from epicycloid curves, are shown in Fig. 16-19.

Fig. 16(b) uses the units of Fig. 16(a), and the color scheme applied was with the same hue. Fig. 16(c) was formed via the combination of the similar figures based on Fig. 16(a), and the color scheme was applied with the gradation of hue. Fig. 17(b) was formed with units of Fig. 17(a), and the color scheme applied was with the same hue. Fig. 17(c) was formed via the combination of the similar figures based on Fig. 17(a), and the color scheme applied was a gradation of hues. Fig. 18(b) uses the units of Fig. 18(a), and the color scheme applied was with the same hue. Fig. 18(c) was formed via the combination of the similar figures based on Fig. 18(a), and the color scheme applied was a gradation of hues. Fig. 19(b) uses the units of Fig. 19(a), and the color scheme applied was with the same hue. Fig. 19(c) was formed via the combination of the similar figures based on Fig. 19(a), and the color scheme applied was a gradation of hues.



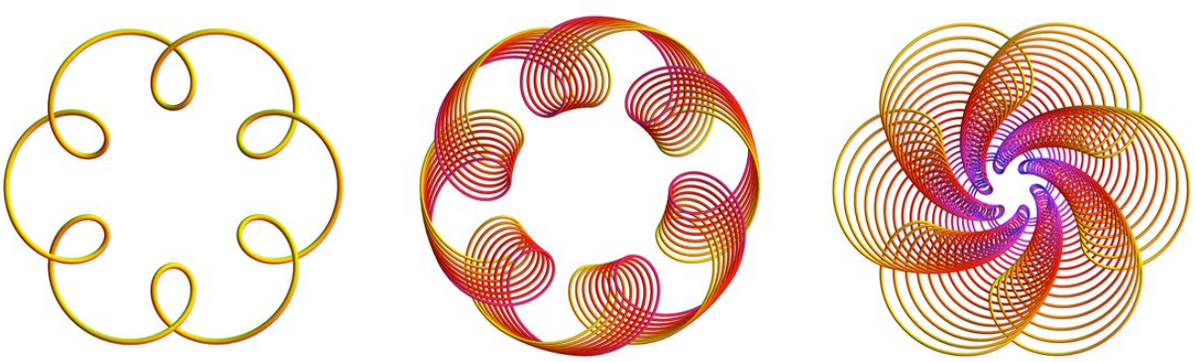
(a) (b) (3, 30°) (c) (13, 7.5°, 0.965)

Figigure. 16 Composition 1 obtained from epitrochoid curves



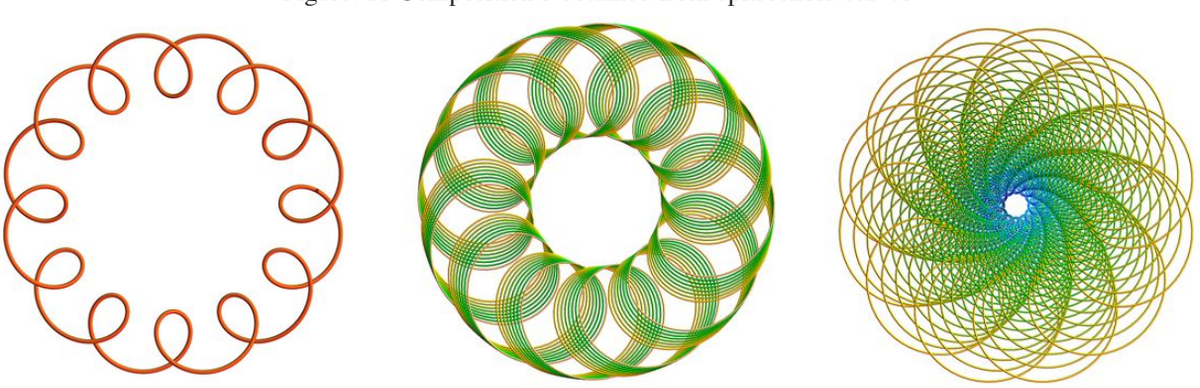
(a) (b) (4, 15°) (c) (19, 1.25°, 0.965)

Figure. 17 Composition 2 obtained from epitrochoid curves



(a) (b) (8, 7.5°) (c) (24, 7.5°, 0.94)

Figure. 18 Composition 3 obtained from epitrochoid curves



(a) (b) (12, 3.75°) (c) (24, 7.5°, 0.93)

Figure. 19 Composition 4 obtained from epitrochoid curves

3.1.4 Composotion of hypotrochoid curves

Hyporochoid curves are defined by the following parametric representation:

$$x = (a-b)\cos t + c\cos (a-b)\, t / b$$
$$y = (a-b)\sin t - c\sin (a-b)\, t / b \qquad (7)$$

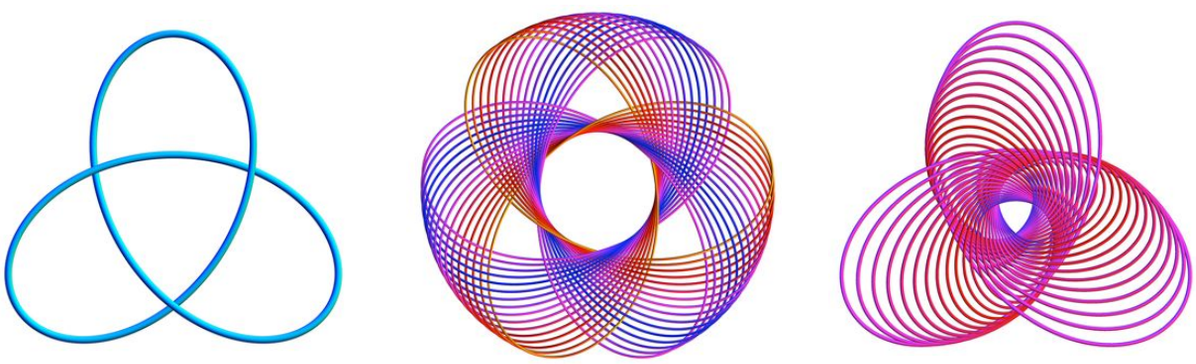
Adding the cordinate z to (7), it becomes:

$$x = (a-b)\cos t + c\cos (a-b)\, t / b$$
$$y = (a-b)\sin t - c\sin (a-b)\, t / b$$
$$z = e\sin f\, t \qquad (8)$$

The geometrical curves are obtained from (8) by varying parameters.

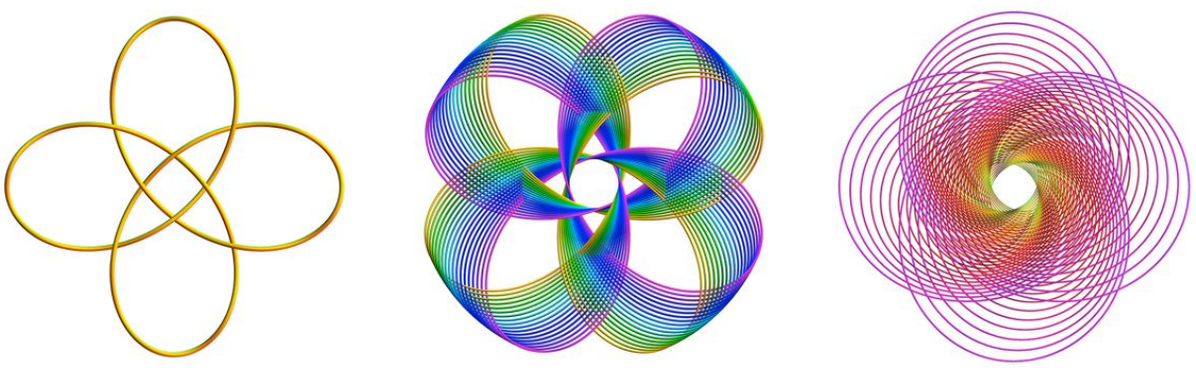
Examples of compositions, obtained from epicycloid curves, are shown in Fig. 20-23.

Fig. 20(b) uses the units of Fig. 20(a), and the color scheme applied was a contrast of hues. Fig. 20(c) was formed via the combination of the similar figures based on Fig. 27(a), and the color scheme applied was a gradation of hues. Fig. 21(b) uses the units of Fig. 21(a), and the color scheme applied was a contrast of hues. Fig. 21(c) was formed via the combination of the similar figures based on Fig. 21(a), and the color scheme applied was a gradation of hues. Fig. 22(b) uses the units of Fig. 22(a), and the color scheme was applied was with the same hue. Fig. 22(c) was formed via the combination of the similar figures based on Fig. 22(a), and the color scheme applied was a gradation of hues. Fig. 23(b) uses units of Fig. 23(a), and the color scheme applied was with the same hue. Fig. 23(c) was formed via the combination of the similar figures based on Fig. 23(a), and the color scheme applied a gradation of hues.



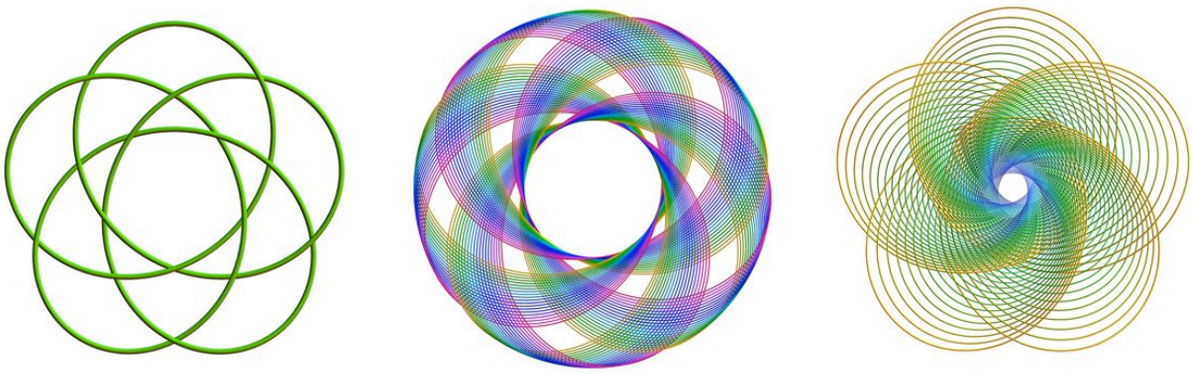
(a) (b) (16, 5°) (c) (25, 5°, 0.938)

Figure. 20 Composition 1 obtained from hypotrochoid curves



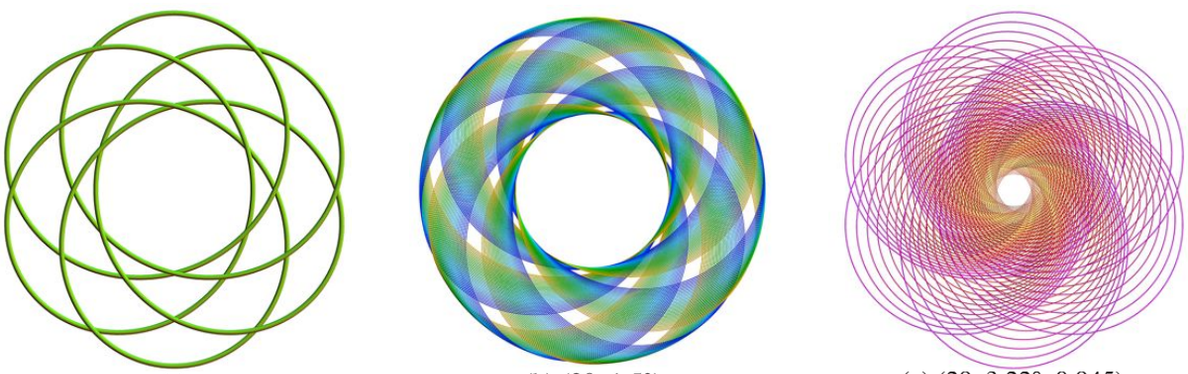
(a) (b) (18, 5°) (c) (25, 3.75°, 0.938)

Figure. 21 Composition 2 obtained from hypotrochoid curves



(a) (b) (20, 5°) (c) (29, 5°, 0.945)

Figure. 22 Composition 3 obtained from hypotrochoid curves



(a) (b) (30, 1.5°) (c) (29, 3.22°, 0.945)

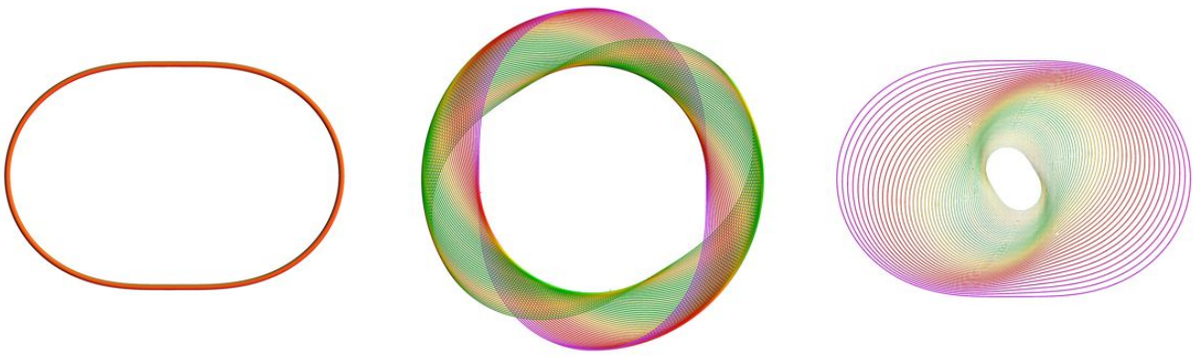
Figure. 23 Composition 4 obtained from hypotrochoid curves

3.1.5 Composotion of torus knots.

Torus Knots are defined by the following parametric representation:

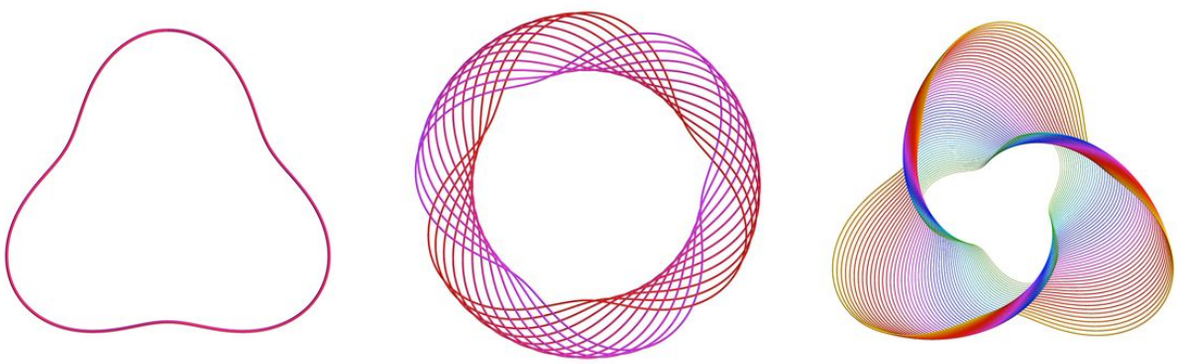
$$x = (a + b \cos c\, t)\cos d\, t$$
$$y = (a + b \cos c\, t)\sin d\, t$$
$$z = e \sin f\, t \qquad (9)$$

The geometrical curves are obtained from (9) by varying parameters. Exmples of compositions obtained from epicycloid curves are shown in Fig. 24-29. Fig. 24(b) uses the units of Fig. 24(a), and the color scheme applied was a contrast of hues. Fig. 24(c) was formed via the combination of the similar figures based on Fig. 24(a), and the color scheme applied was a gradation of hues. Fig. 25(b) uses the units of Fig. 24(a), and the color scheme applied was the same hue. Fig. 25(c) was formed via the combination of the similar figures based on Fig. 24(a), and the color scheme applied was a gradation of hues. Fig. 26(b) uses the units of Fig. 26(a), and the color scheme applied was a gradition of hues. Fig. 26(c) was formed via the combination of the similar figures based on Fig. 26(a), and the color scheme applied was a gradation of hues. Fig. 27(b) uses the units of Fig. 27(a), and the color scheme applied was a contrast of hues. Fig. 27(c) was formed via the combination of the similar figures based on Fig. 27(a), and the color scheme applied was a gradation of hues. Fig. 28(b) uses the units of Fig. 28(a), and the color scheme applied was a contrast of hues. Fig. 28(c) was formed via the combination of the similar figures based on Fig. 28(a), and the color scheme applied was a gradation of hues. Fig. 29(b) uses the units of Fig. 29(a), and the color scheme applied was a contrast hues. Fig. 29(c) was formed via the combination of the similar figures based on Fig. 29(a), and the color scheme was applied was a gradation of hues.



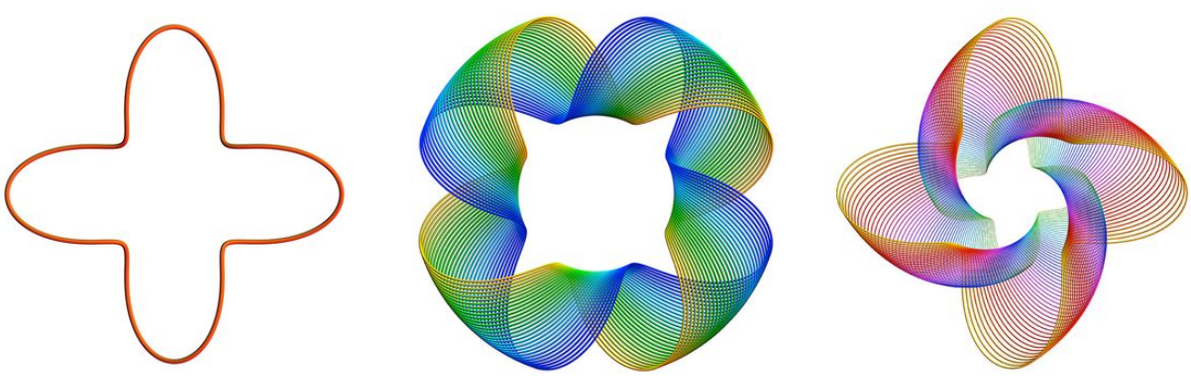
(a) (b) (55, 2.25°) (c) (55, 1°, 0.97)

Figure. 24 Composition 1 obtained from torus knots



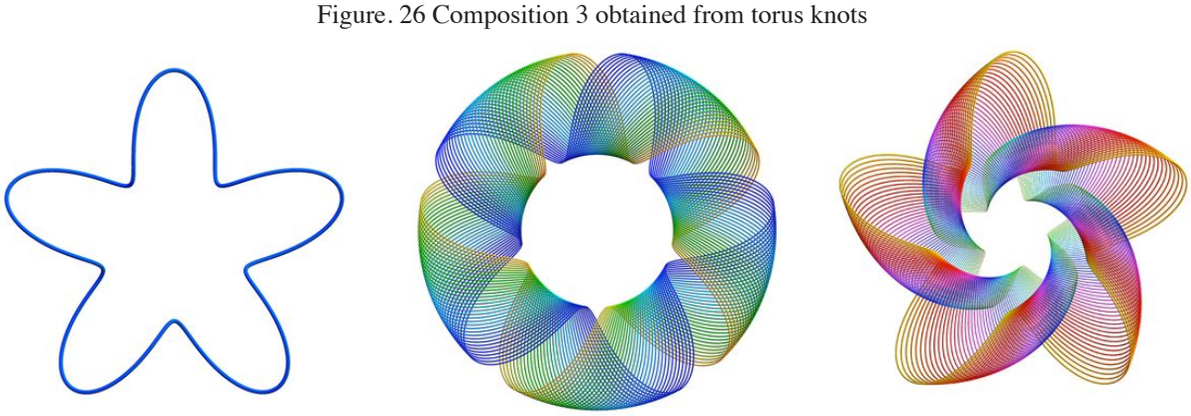
(a) (b) (12, 7.5°) (c) (55, 1.6°, 0.985)

Figure. 25 Composition 2 obtained from torus knots



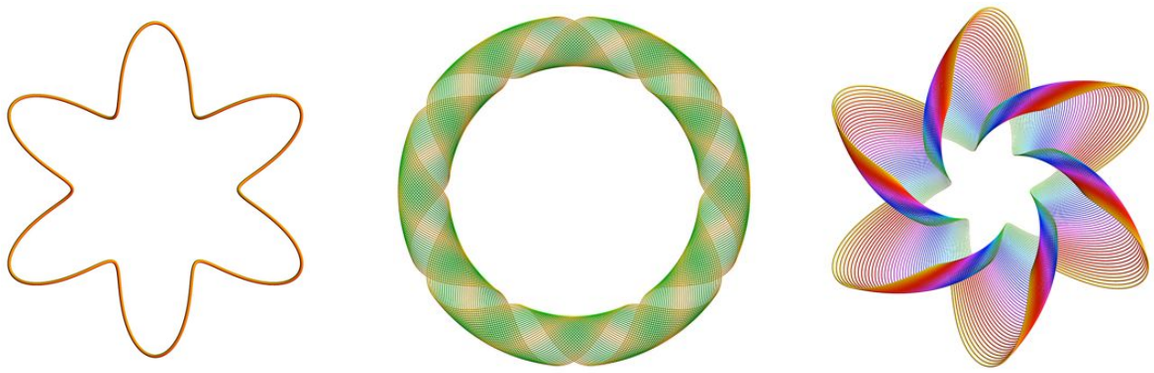
(a) (b) (30, 1.8°) (c) (55, 1.8°, 0.985)

Figure. 26 Composition 3 obtained from torus knots



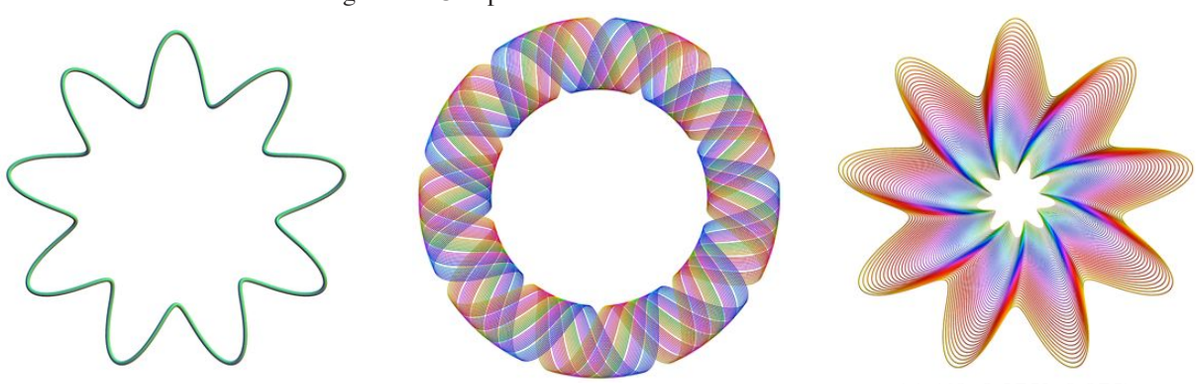
(a) (b) (35, 1.6°) (c) (55, 1.6°, 0.985)

Figure. 27 Composition 4 obtained from torus knots



(a) (b) (40, 1.2°) (c) (55, 1°, 0.985)

Figure. 28 Composition 5 obtained from torus knots



(a) (b) (45, 0.63°) (c) (60, 0.375°, 0.975)

Figure. 29 Composition 6 obtained from torus knots

4.Conclusions

This paper focuses on geometrical curves, using computer graphics, and represents them in an attempt to show different variations of their composition.

The results are:

(1) regarding the form composition based on combinations of basic units, interesting figures appear by overlapping the units.

(2) regarding the form composition based on combinations of similar figures, various figures occur, depending on the basic units and their combinations.

(3) regarding the color compositions, with one hue, gradation of hues, and contrast of hues, they were all successful.

5.References

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